

# POLYMERIZATION KINETICS

## THERMODYNAMICS

*Tells us where the system would like to go **eventually** !!  
ie. defines relationships between macroscopic variables at equilibrium*

## KINETICS

*Tells us **how fast** the system takes various reaction paths*

## EXAMPLES

SUGAR + OXYGEN  $\longrightarrow$  PRODUCTS + ENERGY

CRYSTALLIZATION IS ALSO A PROCESS CONTROLLED BY KINETICS, AS WE WILL SEE LATER

# POLYMERIZATION KINETICS

STEP GROWTH -

SLOW

*Can use statistical methods  
as well as kinetics to describe  
mol. wt. distributions - more  
on this later*

CHAIN Polymerization -

FAST

*Can apply statistical methods  
to an analysis of the  
microstructure of the products,  
but not the polymerization  
process and things like mol .wt.*

# KINETICS OF STEP GROWTH POLYMERIZATION

## WHY BOTHER ?

- *How long does it take to make polymer ?*
- *Can we speed up the reaction ?*
- *What is the relationship between kinetics and the Mol. Wt. Of the product ?*

# REVISION

$$\text{RATE OF REACTION} = \text{CONSTANT} \times [\text{CONCENTRATION}]^n \text{TERMS}$$

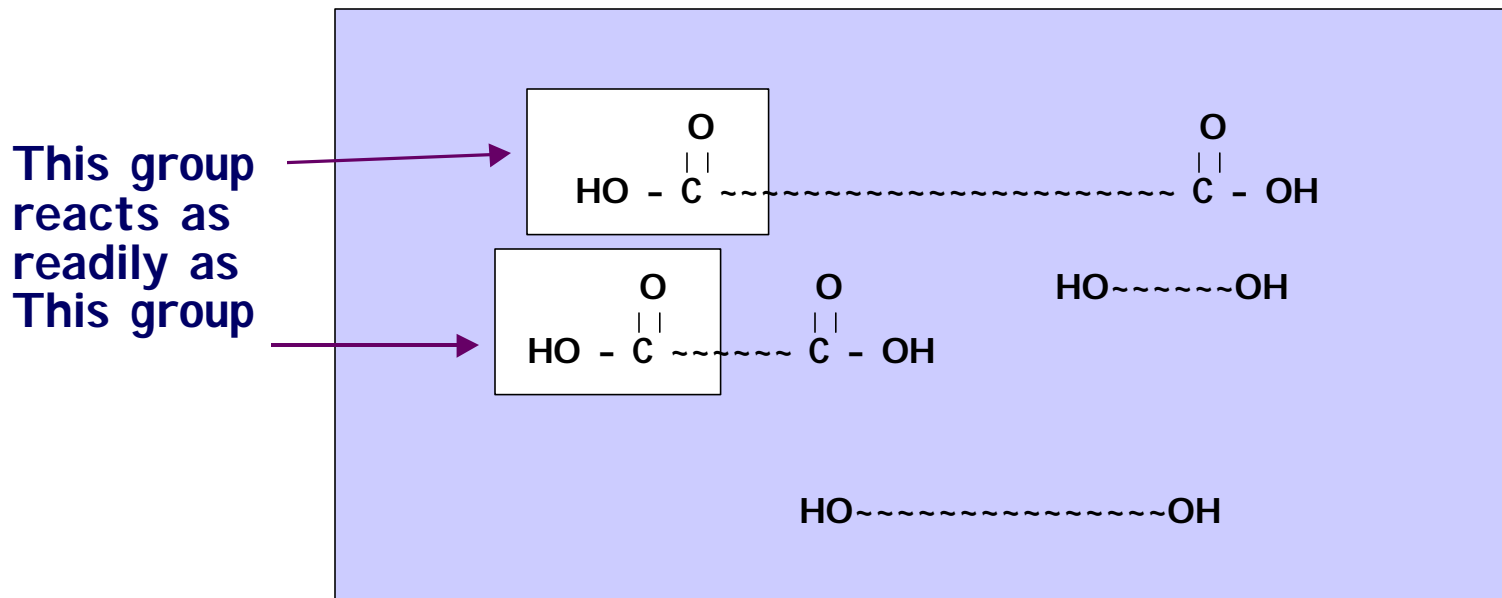
$$\begin{aligned} \text{RATE OF DISAPPEARANCE OF MONOMER} &= - \frac{dM}{dt} \\ &= k \times [\text{CONCENTRATION}]^n \text{TERMS} \end{aligned}$$

# KINETICS OF POLYCONDENSATION

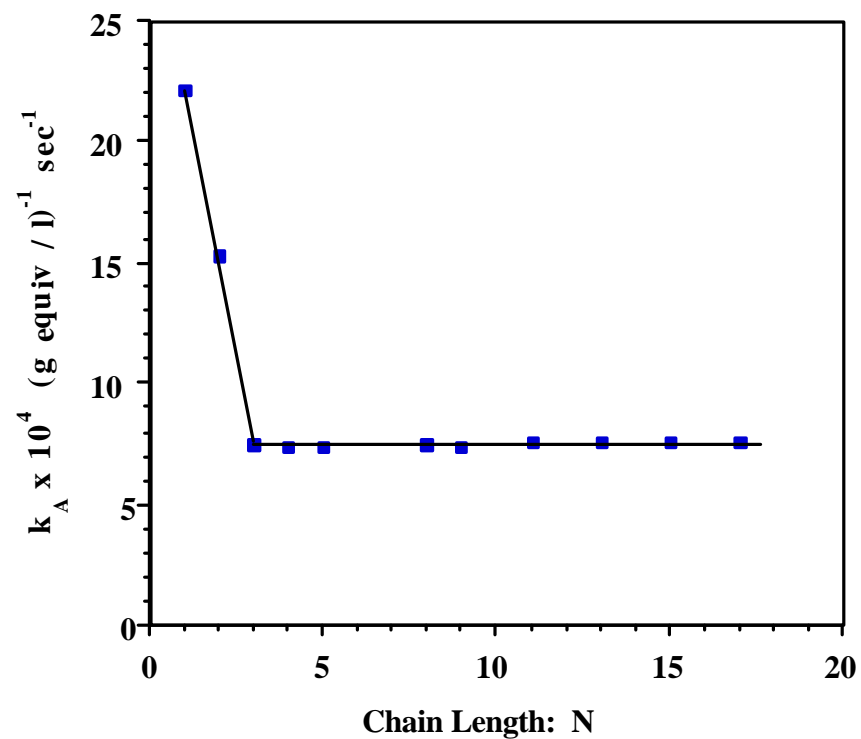
## KEY ASSUMPTION - FLORY

The reactivity of a functional group is independent of the length of the chain to which it is attached

### EXAMPLE:



# WAS FLORY RIGHT?



Redrawn from the data of Flory, P.J.,  
Principles of Polymer Chemistry, Cornell  
University Press, 1953, p71

# KINETICS OF POLYCONDENSATION



*Kinetic equation for this type of reaction is usually of the form:*

$$\text{Reaction Rate} = -\frac{d[A]}{dt} = k_2[A][B]$$

**N.B. [A] AND [B] ARE THE CONCENTRATIONS OF FUNCTIONAL GROUPS**

**However, esterifications are acid catalyzed and *in the absence of added strong acid***

$$-\frac{d[A]}{dt} = k_3[A]^2[B]$$

# MORE KINETICS

$$-\frac{d[A]}{dt} = k_3[A]^2[B]$$

**If**  $[A] = [B]$

$$c = [A] = [B]$$

**Hence**

$$-\frac{dc}{dt} = k_3 c^3$$

$$\int_{c_0}^c \frac{dc}{c^3} = k_3 \int_{t=0}^t dt$$

$$2k_3 t = \frac{1}{c^2} - \frac{1}{c_0^2}$$



# EXTENT OF REACTION

Define

$$p = \text{EXTENT OF REACTION}$$

In this example

$$\text{LET } p = \frac{\text{\# OF COOH GROUPS REACTED}}{\text{\# OF COOH GROUPS ORIGINALLY PRESENT}}$$

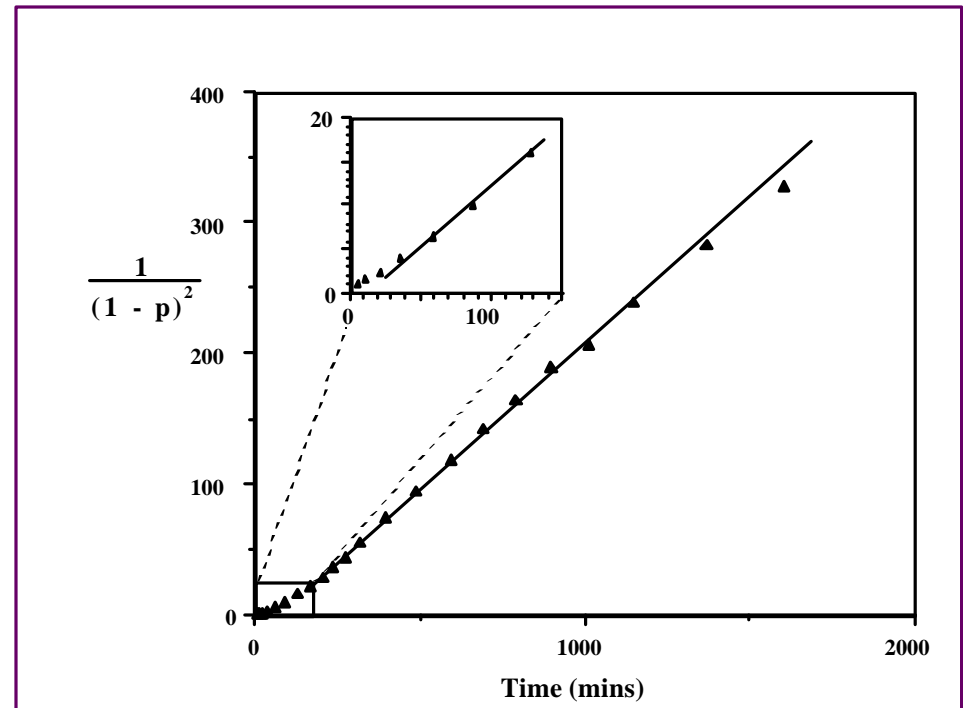
Then

$$c = c_0 (1-p)$$

And

$$2 c_0^2 k_3 t = \frac{1}{(1-p)^2} - 1$$

Redrawn from the data of Flory, P.J.,  
J.A.C.S., 61, 3334 (1939)



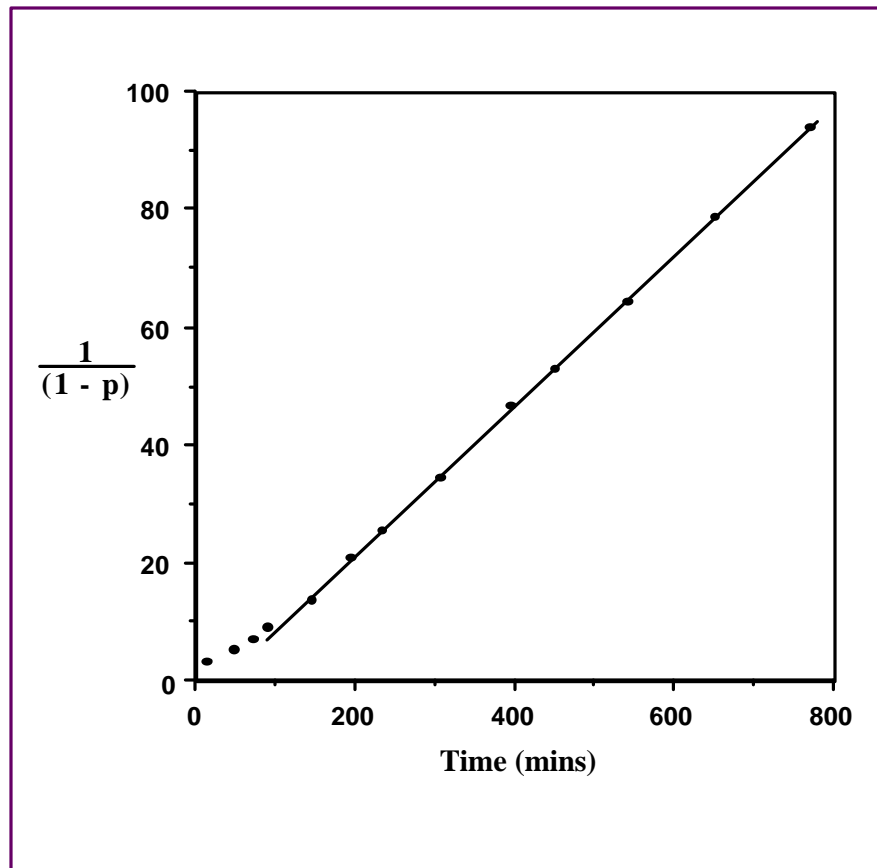
# ACID CATALYZED REACTION

$$-\frac{d[\text{COOH}]}{dt} = k' [\text{COOH}] [\text{OH}]$$

$$-\frac{dc}{dt} = k'c^2$$

$$c_0 k' t = \frac{1}{(1-p)} + \text{constant}$$

*Note; the concentration of the acid catalyst (a constant) is included in  $k'$*



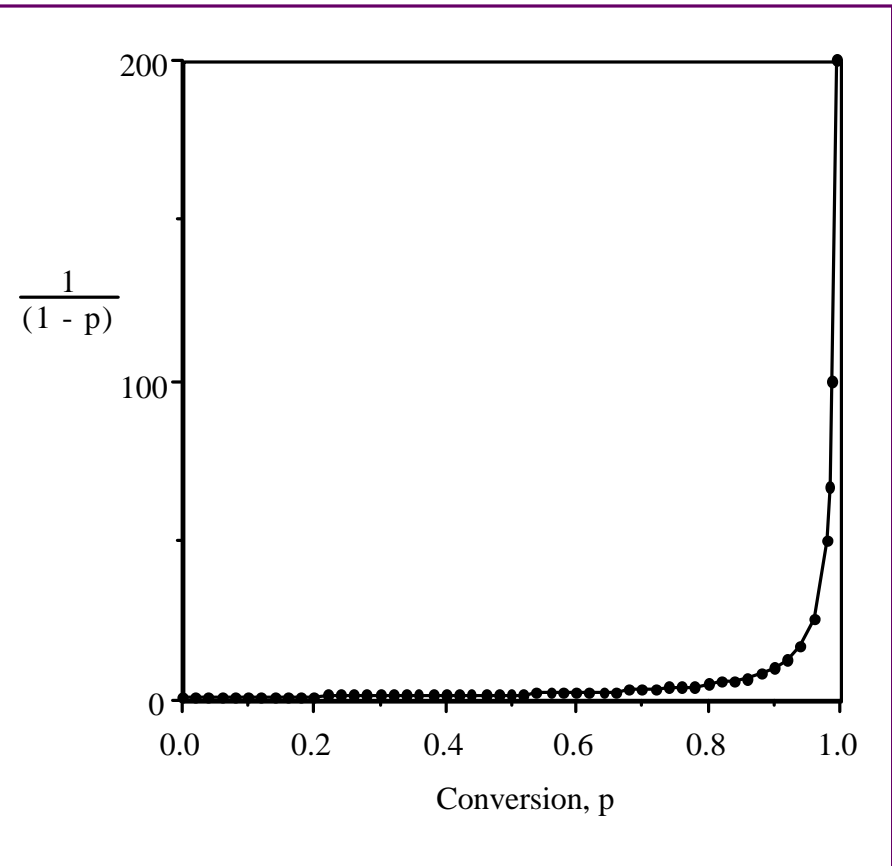
Redrawn from the data of Flory, P.J.,  
J.A.C.S., 61, 3334 (1939)

# NUMBER AVERAGE DEGREE OF POLYMERIZATION

$$\begin{aligned}\bar{x}_n &= \frac{N_0}{N} \\ &= \frac{c_0}{c} = \frac{c_0}{c_0(1-p)}\end{aligned}$$

ie  $\bar{x}_n = \frac{1}{(1-p)}$

$$\bar{M}_n = M_0 \bar{x}_n = \frac{M_0}{(1-p)}$$

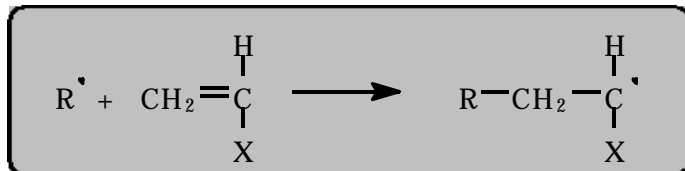
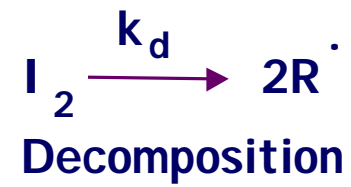
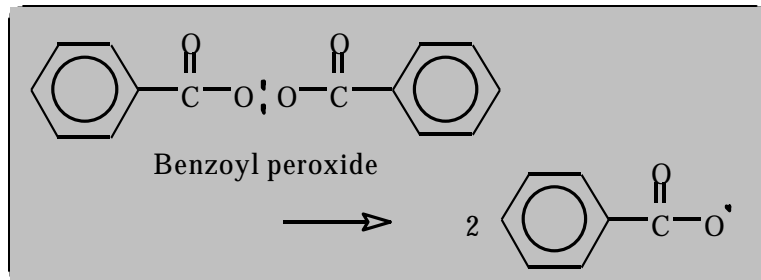


# **KINETICS OF FREE RADICAL POLYMERIZATION**

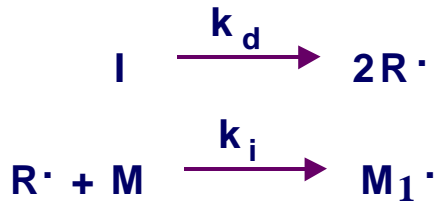
*We need to consider the following steps*

- **INITIATION**
- **PROPAGATION**
- **(CHAIN TRANSFER)**
- **TERMINATION**

# INITIATION



# INITIATION



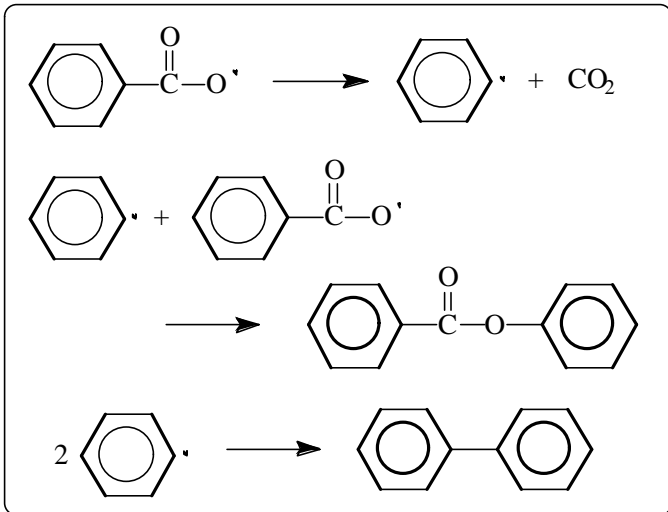
ASSUME DECOMPOSITION IS THE RATE LIMITING STEP

i.e.  $k_i \gg k_d$

THEN WE SHOULD ONLY HAVE TO CONSIDER  $k_d$

$$\text{i.e. } -\frac{d[I]}{dt} = \frac{1}{2} \frac{d[M_1\cdot]}{dt} = k_d [I]$$

**BUT; ONLY A FRACTION  $f$  OF RADICALS INITIATE CHAIN GROWTH**



$$r_i = \frac{d[M_i]}{dt} = 2 f k_d [I]$$

# PROPAGATION



In general;

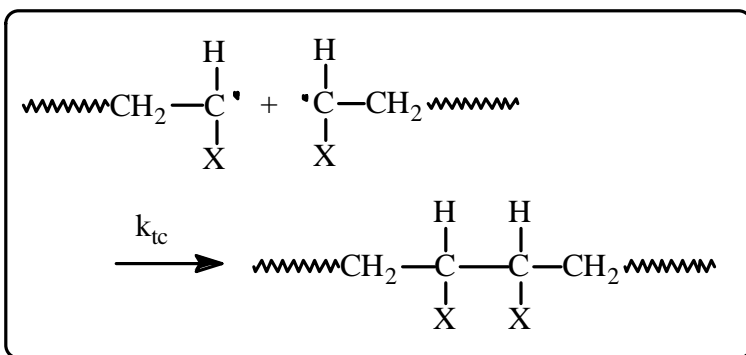


$$r_p = -\frac{d[M]}{dt} = k_p [M][M\cdot]$$

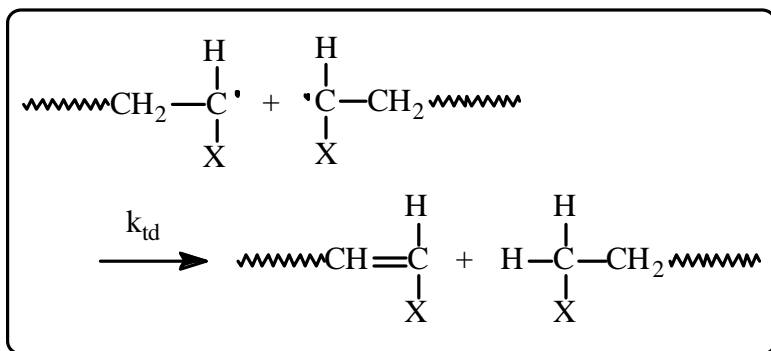
Assumption; reactivity is independent of chain length

# TERMINATION

## COMBINATION



## DISPROPORTIONATION





# RATE OF TERMINATION

$$r_t = -\frac{d[M\cdot]}{dt} = 2 k_t [M\cdot][M\cdot]$$

Where  $k_t = k_{tc} + k_{td}$

OBTAINED FROM :

- *Both reactions are second order*
- *Rate of removal of chain radicals = sum of the rates of the two termination reactions*

# SUMMARY

$$r_i = \frac{d[M_i]}{dt} = 2 f k_d [I]$$

$$r_p = -\frac{d[M]}{dt} = k_p [M][M\cdot]$$

$$r_t = -\frac{d[M\cdot]}{dt} = 2 k_t [M\cdot][M\cdot]$$

PROBLEM :

*We don't know [M·]*

SOLUTION :

*Assume a steady state concentration of transient species*

# STEADY STATE ASSUMPTION

$$[ M\cdot ] = \text{CONSTANT}$$

*This means that radicals are consumed at the same rate as they are generated*

$$r_i = r_t$$

$$2f k_d [ I ] = 2 k_t [ M\cdot ]^2$$

$$[ M\cdot ] = \left[ \frac{fk_d [ I ]}{k_t} \right]^{1/2}$$

# RATE OF PROPAGATION

RATE OF PROPAGATION = RATE OF POLYMERIZATION

$$r_p = R_p$$

*substituting;* 
$$r_p = k_p \left[ \frac{fk_d [ I ]}{k_t} \right]^{1/2} [ M ]$$

BUT [ I ] IS NOT CONSTANT

*from* 
$$-\frac{d[ I ]}{dt} = k_d [ I ] \text{ obtain } [ I ] = [ I_0 ] e^{-k_d t}$$

HENCE

$$R_p = \left[ k_p \left[ \frac{f k_d}{k_t} \right]^{1/2} [ M ] [ I_0 ]^{1/2} \right] \left[ e^{-k_d t/2} \right]$$

# WHAT DOES THIS TELL US ?

1. IF WE WANT TO INCREASE  $R_p$   
INCREASE  $[M]$  OR  $[I]$

*But; changing  $[I]$  also changes  
mol. wt. - more on this later !*

2.  $R_p \sim k_p / k_t^{1/2}$

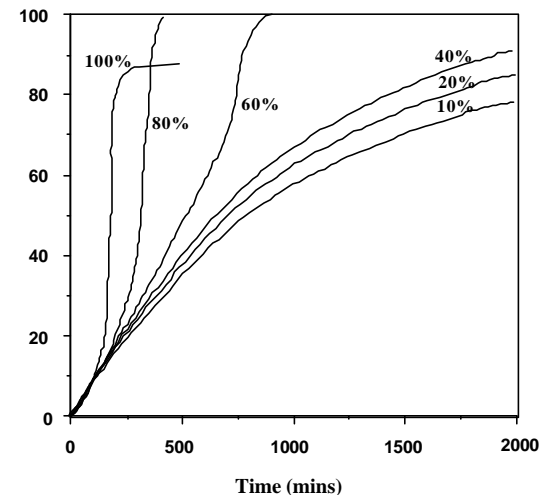
*For ethylene at 130 °C and  
1 bar pressure*

$$k_p / k_t^{1/2} \sim 0.05$$

*For ethylene at 200 °C and  
2500 bar pressure*

$$k_p / k_t^{1/2} \sim 3.0$$

3. TROMSDORFF EFFECT



# CONVERSION

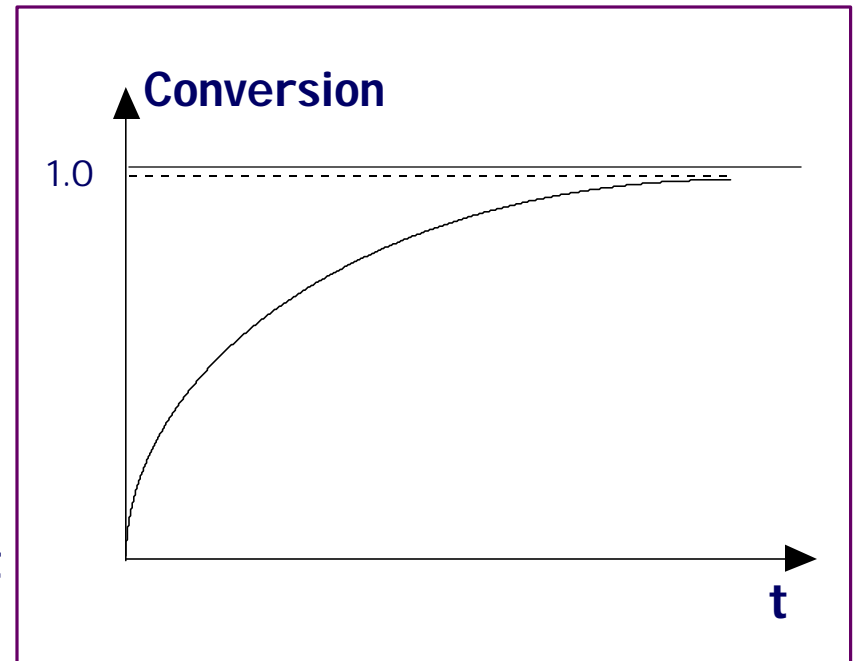
DEFINITION ;  $\frac{[M_0] - [M]}{[M_0]} = \frac{\text{Amount of monomer used up}}{\text{Amount of monomer at start}}$

*in initial stages of reaction we can assume  $[I] = [I_0] = \text{constant}$*

$$\frac{d[M]}{dt} = -k_p \left[ \frac{fk_d[I]}{k_t} \right]^{1/2} [M]$$

*Integrating*

$$\ln \frac{[M]}{[M_0]} = -k_p \left[ \frac{fk_d[I_0]}{k_t} \right]^{1/2} t$$



# MAXIMUM CONVERSION

USUALLY THERE IS A FIRST ORDER DECAY IN INITIATOR CONCENTRATION

$$\text{i.e. } \frac{d[I]}{dt} = -k_d[I] \quad [I] = [I_0] e^{-k_d t}$$

and

$$\ln \frac{[M]}{[M_0]} = - \frac{2 k_p}{k_d} \left[ \frac{f k_d [I_0]}{k_t} \right]^{1/2} [1 - e^{-k_d t}]$$

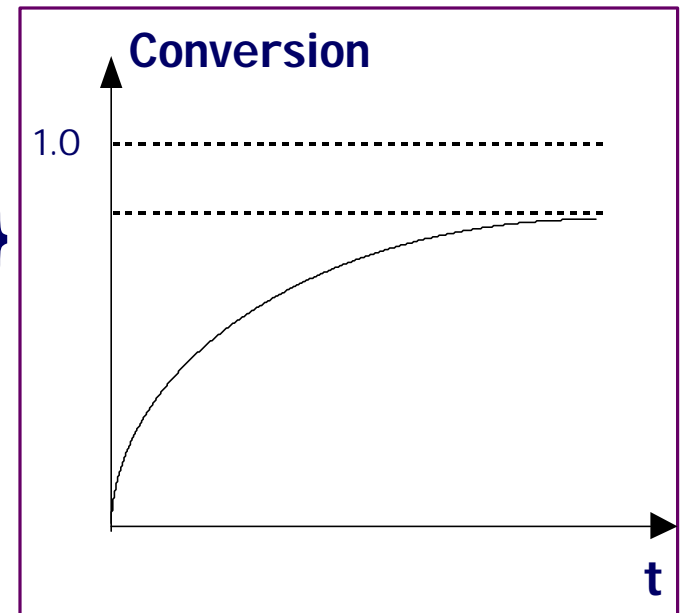
hence

CONVERSION =

$$1 - \exp - \left\{ 2 k_p \left[ \frac{f [I_0]}{k_t k_d} \right]^{1/2} [1 - e^{-k_d t/2}] \right\}$$

MAX CONVERSION (  $t \rightarrow \infty$  ) =

$$1 - \exp - \left\{ 2 k_p \left[ \frac{f [I_0]}{k_t k_d} \right]^{1/2} \right\}$$



# AVERAGE CHAIN LENGTH

DEFINE *KINETIC CHAIN LENGTH*

$$\bar{n} = \frac{\text{RATE OF MONOMER ADDITION TO GROWING CHAINS}}{\text{RATE AT WHICH CHAINS ARE STARTED}}$$

*This is the average number of monomers polymerized per chain radical at a particular instant of time during the polymerization*



# KINETIC CHAIN LENGTH

CONSIDER A TIME PERIOD  $t$

*let us say that*

- 1. 100 chains are started*
- 2. 1,000,000 monomers are reacted in this time period*

*Then the average degree of polymerization of these chains is*  $\longrightarrow \frac{1,000,000}{100} = 10,000$

# KINETIC CHAIN LENGTH

THERE WILL BE SOME OBVIOUS ERRORS (e.g. What about chains that were initiated, but did not terminate just before the start of the chosen period ?), BUT THESE DECREASE AS  $t \rightarrow$  SMALL

IN THE LIMIT OF A TIME PERIOD  $dt$

$$\bar{n} = \frac{r_p}{r_i} = \frac{k_p [M]}{2 (f k_d k_t)^{1/2} [I]^{1/2}}$$

$$\text{i.e. } \bar{n} \sim \frac{[M]}{[I]^{1/2}} \quad \text{c.f. } r_p \sim [M][I]^{1/2}$$

THE DEGREE OF POLYMERIZATION THEN DEPENDS UPON THE MECHANISM OF TERMINATION

$$\begin{aligned} \bar{x}_n &= \bar{n} & - \text{disproportionation} \\ \bar{x}_n &= 2 \bar{n} & - \text{combination} \end{aligned}$$

# INSTANTANEOUS NUMBER AVERAGE CHAIN LENGTH

*What if termination occurs by both mechanisms ? define an average number of dead chains per termination reaction*

$$\begin{aligned}
 \bar{x} &= \frac{\text{rate of dead chain formation}}{\text{rate of termination}} \\
 &= \frac{[2 k_{td} + k_{tc}] [M\cdot]^2}{[k_{td} + k_{tc}] [M\cdot]^2} = \frac{[2 k_{td} + k_{tc}]}{k_t}
 \end{aligned}$$

HENCE

$$\begin{aligned}
 \bar{X}_n &= \frac{k_p [M\cdot] [M]}{[2 k_{td} + k_{tc}] [M\cdot]^2} \\
 \bar{X}_n &= \frac{k_p [M]}{\bar{x} [f k_d k_t]^{1/2} [I]^{1/2}}
 \end{aligned}$$

# CHAIN TRANSFER

Chain transfer can occur to solvent, added agents, etc.



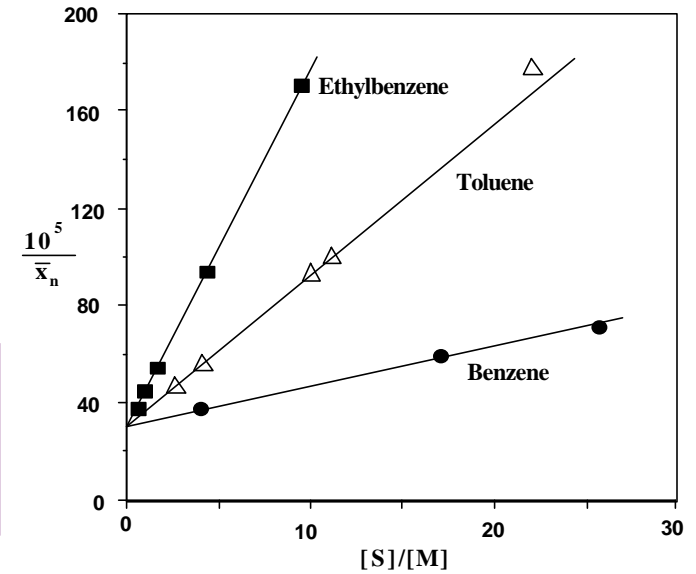
$$-\frac{d[M \cdot]}{dt} = 2k_{td} [M \cdot]^2 + k_{tc} [M \cdot]^2 + k_{tr} [T] [M \cdot]$$

CAN THEN OBTAIN

$$\bar{x}_n = \frac{k_p [M]}{k_t [f k_d k_t]^{1/2} [I]^{1/2} + k_{tr} [T]}$$

OR

$$\frac{1}{\bar{x}_n} = \frac{1}{(\bar{x}_n)_0} + \frac{C [T]}{[M]} \quad \text{where } C = k_{tr} / k_p$$



Redrawn from the data of R.A.Gregg + F.R.Mayo, Faraday Soc. Discussions, 2, 328 (1947)